Name:___________________________ Date:_________________________

M10/11H HW Section 4.9 Part 2 Double Angle Identities

$\sin(a-b)=\sin a\cos b-\sin b\cos a$ $\cos(a+b)=\cos a\cos b-\sin b\sin a$ $\sin(a+b)=\sin a\cos b+\sin b\cos a$

$\cos(a-b)=\cos a\cos b+\sin b\sin a$ $\tan(a+b)=\dfrac{\tan a+\tan b}{1-\tan a\tan b}$ $\tan(a-b)=\dfrac{\tan a-\tan b}{1+\tan a\tan b}$

$\sin 2a = 2\sin a\cos a$ $\cos 2a=\cos^2 a-\sin^2 a$ $\cos 2a = 1-2\sin^2 a$ $\cos 2a = 2\cos^2 a - 1$

1. How do we derive the equation $\cos 2\theta=\cos^2\theta-\sin^2\theta$ from $\cos(a+b)=\cos a\cos b-\sin a\sin b$?

2. How do we derive the equation $\sin 2\theta = 2\sin\theta\cos\theta$ from the identity $\sin(a+b)=\sin a\cos b+\sin b\cos a$?

3. When proving an identity that involves $\cos 2\theta$, how do we tell which of the three equations to use? Should we convert it to $\cos^2\theta-\sin^2\theta$, $1-2\sin^2\theta$, OR $2\cos^2\theta-1$? Are they the same?

4. Prove that $\cos 2\theta = 1-2\sin^2\theta$ and $\cos 2\theta = 2\cos^2\theta - 1$

5. Can we simplify the equation like this? Explain why or why not:

$$\frac{\sin 2\theta}{2}=\frac{\sin \cancel{2}\theta}{\cancel{2}}=\sin\theta$$

6. Use the double angle identities to simplify each of the following to a single trigonometric function. Show all your work and steps:

a) $2\sin 0.4\cos 0.4$	b) $2\sin 5\cos 5$
c) $\cos^2 0.7-\sin^2 0.7$	d) $2\sin\frac{\pi}{7}\cos\frac{\pi}{7}$
e) $\cos^2\frac{\pi}{9}-\sin^2\frac{\pi}{9}$	f) $2\cos^2 75°-1$

7. Given that $\sin\theta=\frac{2}{3}$, where θ is in Q1, find the exact value of the following: $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

8. Given that $\tan\theta=-\frac{5}{6}$, where θ is in Q2, find the exact value of the following: $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$

9. Solve the following equation for $0\le\theta\le 2\pi$ using sum and difference identities:

a) $\sin\left(\theta+\frac{\pi}{2}\right)=\frac{1}{2}$	b) $\cos\left(\frac{3\pi}{2}+\theta\right)+\sin\left(\frac{3\pi}{2}-\theta\right)=0$
c) $\sin\left(\theta+\frac{\pi}{3}\right)+\sin\left(\theta-\frac{\pi}{3}\right)=1$	d) $\cos\left(\theta-\frac{\pi}{6}\right)+\sin\left(\theta-\frac{\pi}{3}\right)=\sin\theta$

10. Prove the following identities:

a) $\sin\left(270^\circ + \theta\right) = -\cos\theta$	b) $\sin\left(2\pi + \theta\right) = \sin\theta$
c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$	d) $\cos\left(a + m\right) - \cos\left(a - m\right) = -2\sin a \sin b$
e) $\cos\left(A + B\right) \bullet \cos B + \sin\left(A + B\right) \bullet \sin B = \cos A$	$\sin\left(x + y\right) + \sin\left(x - y\right) = 2\sin x \cos y$

11. Prove the following identities:

a) $\sin\left(3\pi - \theta\right) = \sin\theta$	b) $\cos\left(a + b\right) + \cos\left(a - b\right) = 2\cos a \cos b$

c) $\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=-\sec 2\theta$	d) $\tan a-\tan b=\frac{\sin(a-b)}{(\cos a)(\cos b)}$
e) $1+\sin 2\theta=(\sin\theta+\cos\theta)^2$	f) $\sin 2\theta=2\cot\theta\sin^2\theta$
g) $\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}$	h) $\sec^2\theta=\frac{2}{1+\cos 2\theta}$

i) $\dfrac{1-\cos 2\theta}{2}=\sin^2\theta$	J) $\dfrac{(\sin\theta+\cos\theta)^2}{\sin 2\theta}=\csc 2\theta+1$
K) $\dfrac{\sin 3x}{\sin x}-\dfrac{\cos 3x}{\cos x}=2$	L) $4\cos^2\theta+\cos(2\theta)=5-6\sin^2 x$
m) $\cos^4 x-\sin^4 x=\cos(2x)$	n) $\cot(2\theta)=\dfrac{1}{2}(\cot\theta-\tan\theta)$

O) $\dfrac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta}=\cos(2\theta)$	P) $\cot^2(2\theta)=\dfrac{\cot^2\theta-1}{2\cot\theta}$
Q) $\cos^2(2\theta)-\sin^2(2\theta)=\cos(4\theta)$	R) $(4\sin\theta\cos\theta)(1-2\sin^2\theta)=\sin(4\theta)$
S) $3\sin^2\theta\cos^2\theta=\dfrac{3}{8}(1-\cos(4\theta))$	T) $\sin 2\theta=2\tan\theta-2\tan\theta\sin^2\theta$